Pergamon

Journal of Structural Geology, Vol. 19, No. 1, pp. 129 to 132, 1997

0191-8141/97 $17.00+0.00

PII: S0191-8141(96)00076-4

Brevia

SHORT NOTICE

Fractal strain distribution and its implications for cross-section balancing: further discussion

THOMAS H. WILSON

Department of Geology and Geography, West Virginia University, Morgantown, WV 26506, U.S.A.

(*Received* 9 *January* 1996; *accepted in revised form* 30 *August* 1996)

INTRODUCTION

The observations of Wu (1993, 1995) and Dunne and Ferrill (1995) indicate that fractal characterization can provide partial solutions and helpful insights into the problems of cross-section balancing. Wu (1993) uses both ruler and spectral methods to compute the fractal dimension (D_c and D_p, respectively) of a structural profile. Wu concludes that D_p is underestimated (Wu, 1993, pp. 1503 and 1504) and attempts to resolve the differences between the spectral and ruler estimates by introducing vertical exaggeration into the profile (Brown, 1987; Wong, 1987). Wu, 1993 (bottom of p. 1503) notes that inherent differences between the ruler and spectral methods may exist and adopts an intermediate value obtained through vertical exaggeration of the profile (Wu, 1993 right side of p. 1504). Wu, 1993 (p. 1506) concludes that vertical exaggeration of the profile is necessary to obtain a useful D_c, but Wu, 1995 (e.g. bottom page 762) employs D_c without rescaling. The following discussion notes inherent differences between D_c and D_p and presents examples of possible errors that can arise in the computation of D_p.

SELF SIMILARITY VERSUS SELF AFFINITY

Existence of the relationship $N_i = C/r_i^D$, defines a self-similar fractal (Mandelbrot, 1985), where N_i is the number of steps required to walk a curve with compass opening r_i, and D is the fractal dimension (D_c). The requirement of self-similarity is implicit in the application of fractals to bed-length balancing.

The compass method of determining fractal dimension (D_c) provides a direct estimate of curve length over a certain range of scales. Within the context of strain estimation, the length of a deformed layer is treated as a self-similar fractal characterized by compass dimension (D_c); otherwise, accurate estimates of the influence of finer scale structure on the length of a profile cannot be obtained.

The notion of self-affinity usually arises in the context of spatial or temporal variation of properties having different units such as the variation of the Earth's magnetic field with time at some point on the Earth's surface (Turcotte, 1992). While length of a self-similar trail remains invariant to rotation of coordinate axes, the idea of length and axis rotation are meaningless for self-affine fractals.

Self-affine fractals have power spectra $S(f)$ that vary as $f^{-\beta}$, where f is frequency and β the slope of the log-log spectrum (Turcotte, 1992). Spectral analysis of a function requires that it be single-valued. Spectral analysis can be conducted on spatial functions such as profile relief (e.g. Wilson, 1989; Wu, 1993). However, the physical significance of the power-spectral measurement of fractal dimension (D_p), is not directly related to curve length, but to the height-height correlation function (Wong, 1987). The height-height correlation is just the autocorrelation of profile height, and the Fourier transform of the autocorrelation function is the power spectrum (Oppenheim and Schafer, 1975).

The power spectral method is truly a scale invariant measure of D since the log-log slope of the power spectrum is invariant to arbitrary rescaling of the input. However, power spectral estimates of D can be quite variable and depend on factors other than the fractal characteristics of a given data set. Sources of error in the power spectral estimate are associated with sampling, noise, edge effects, and statistical nonstationarity. Wu, 1993 (p. 1502) assumes that faults and overturned folds are absent, or of minor importance along the horizon of interest. If this assumption is incorrect additional error is introduced and the assumption of self-affinity cannot be made.

SOURCES OF ERROR IN D_P

Wu (1993) evaluated the fractal characteristics of structural relief along the Silurian Wills Creek Formation in the section of Dean *et al.* (1985) shown here as Segment

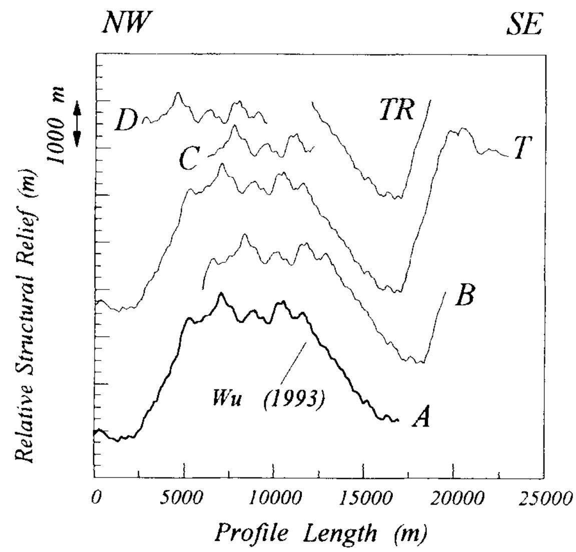

Fig. 1. Structural relief along the base of the Silurian Wills Creek Formation digitized from the section of Dean *et al.* (1985) is shown as profile T. Segment A is the portion of the profile examined by Wu (1993). Analysis of additional subdivisions of the profile (B, C, D, and TR) are presented in the discussion.

A of Fig. 1. For this discussion, that horizon was digitized at sample intervals of 134 m, 128 m, and 48 m (outcrop scale). Computation of D_p from the slope of the power spectrum for these sample intervals yields 1.18 ± 0.085, 1.145 ± 0.075, and 1.13 ± 0.045, respectively.

Additional error is present in the estimates when the profile is sampled at an interval smaller than the wavelengths of structures actually represented in the profile. For example, a 48 m sample interval will reveal folds along the profile with wavelengths of 96 m. However, folds with wavelengths of less than 250 m do not appear in the interpretation presented by Dean *et al.*

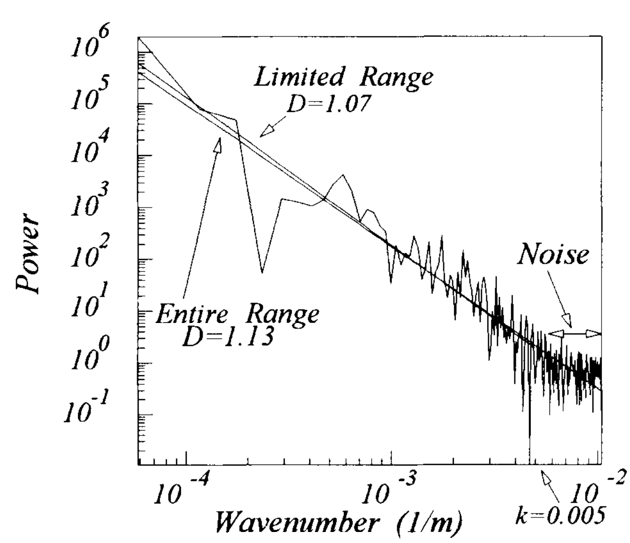

Fig. 2. Log–log plot of the power spectrum for Segment A (Fig. 1) computed at a 48 m sample interval. Regression lines corresponding to fractal dimensions of 1.07 and 1.13 computed for $k \leq 0.005\ \mathrm{m}^{-1}$ and $k \leq 0.004\ \mathrm{m}^{-1}$, respectively are plotted for comparison.

(1985). This leads to flattening in the higher wavenumber region of the spectrum (wavenumbers of $0.004\ \mathrm{m}^{-1}$ and greater in Fig. 2). The break in slope and the realization that these small wavelengths (higher wavenumbers) are unrelated to actual structure, suggests that inclusion of this high wavenumber region of the spectrum in the computation of D_p will introduce error into the estimate. Also, choice of different cut-off frequencies will result in different values of D. For example, D_p computed from wavenumbers less than $0.005\ \mathrm{m}^{-1}$ (see Fig. 2), is 1.07 ± 0.08, while D_p computed for wavenumbers less than $0.004\ \mathrm{m}^{-1}$ yields a D_p of 1.16 ± 0.08. Oversampling offers a means to discriminate between signal and noise. The accuracy of the D_p may be improved by confining the computations to the range of wavelengths actually represented in the data. However, if the break between 'signal' and noise is misidentified, additional error may be introduced as illustrated in this example.

The different fractal dimensions calculated for Segment A (Fig. 1) are tabulated below for comparison (Table 1). D_p of 1.105 obtained by Wu, 1993 (p. 1503) could easily be obtained through minor differences in digitization, sampling interval, and the actual start and end points of the segment analyzed.

Error in the estimate of D_p may also be introduced in the form of edge effects and internal discontinuities. Figure 3(a) and (b) represent two non-fractal functions, a linear rise and step discontinuity, respectively. Both these functions have Fourier series coefficients that vary as $1/f$, so that their power (amplitude squared) varies as $1/f^2$. Hence, the slope of their power spectra, β, will be 2, and D_p, 1.5.

The geologic analogue of the linear-rise is a gradual rise in structural level across a profile. The region of analysis could lie on the flank of a larger structure, for example. Examination of the entire profile (Fig. 1, curve T) reveals an increase in the structural relief of the major anticlines to the southeast as shallow structures in the roof sequence are thrust upward above a Cambrian–Ordovician horse beneath Great North Mountain. Not surprisingly, the power spectrum of the entire profile yields a D_p of 1.64 ± 0.015 (48 m sample interval). The presence of a rise in structural relief across the profile is the certain cause of the high fractal dimension obtained for the whole profile. The geologic analogue of the step-

Table 1. Power spectral estimates of fractal dimension (D_p) for Segment A calculated for different sample rates and different wavenumber ranges (k)

Δx (m)	k-range	D_p
48	total	1.131
48	$\leq$0.005 m	1.07
48	$\leq$0.004 m	1.16
122	total	1.145
134	total	1.18

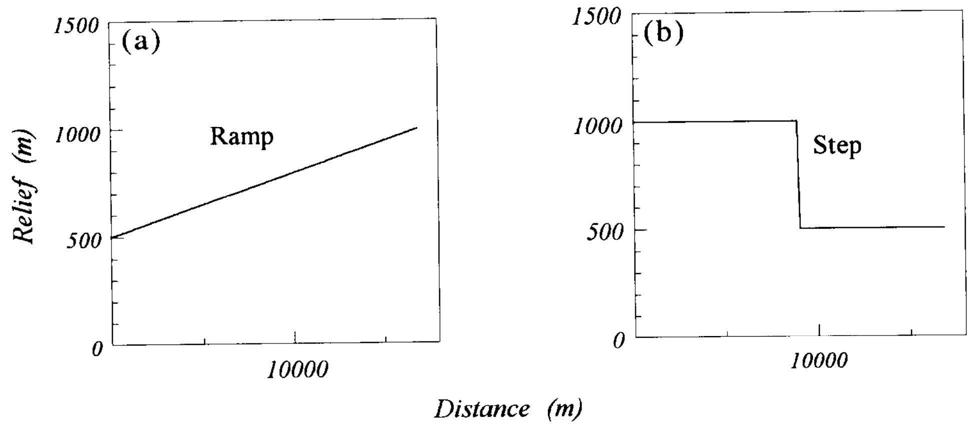

Fig. 3. Simplified, non-fractal, profiles consisting of (a) a linear rise in slope, and (b) a vertical step discontinuity have power spectral slopes (β) equal to 2.

discontinuity (Fig. 3b) occurs in the form of a fault step in an otherwise undeformed layer. Although normal offsets are not present along this profile (Fig. 1), the model serves to illustrate a source of error that might arise in an extensional tectonic setting. D_p of 1 implies that the curves are self-similar. The step and the ramp are self-similar and have compass dimension D_c of 1.

D_P VERSUS D_R

Structural relief along the base of the Silurian Wills Creek Formation has been divided into segments T (entire profile), A, B, C, D, and TR (Fig. 1). D_p and D_R were computed for each segment (Table 2). Based on the preceding discussion, significant variation in the values of D_p are expected.

The values obtained for D_p vary between 0.9 and 1.6. Their variation is related primarily to edge effects. Nonstationarity in the structural characteristics of the different segments (Fig. 1) is also considered a possible source of error. For example, the wavelengths and amplitudes of the folds represented in Segments C and D are significantly different from those interpreted for Segment TR. The structural (and statistical) characteristics of the two regions are quite different. Large segments (Segments A, B, and T) include different proportions of these different provinces. The combination of nonstationarity and edge effects obscures any consistency or structural interrelationship between the various values of D_p listed in Table 1. The results provide no useful geologic information.

Table 2. Power-spectral and compass estimates of D_p and D_c are tabulated for several segments of the structural profile (Fig. 1). The estimates were made on data sampled at 48 m intervals. Wavenumber greater than $0.004\ m^{-1}$ (wavelengths of 250 m) were not incorporated in the computation of D_p

Segment	D_p	D_c
T	1.6	1.041
A	1.16	1.029
B	0.84	1.037
C	1.17	1.033
D	1.19	1.03
TR	0.9	1.038

Fractal dimensions determined from compass measurements (D_c) vary by approximately 1% or less. Although these variations are small, the differences appear consistent with the structural elements present in each segment. For example, the highest D_c (1.041) is associated with the entire line. The large amplitude steep-limbed structure on the eastern end of the profile, if representative of structural variability over a certain range of scales, would yield larger shortening estimates over that range. D_c for Segment TR may seem anomalously high, but if shortening is accommodated by this tighter, high-relief, style of fold, then greater shortening would be expected over an extended range of scales. Segment D ($D_c = 1.03 \pm 0.0013$) covers a slightly longer interval than Segment C ($D_c = 1.033 \pm 0.0014$). The possibility that the fractal dimension for Segment D is slightly higher than that for Segment C, also seems reasonable, since the added intervals incorporate lower relief structures into Segment D. The added features, if representative of all scales, reduce the tendency of the curve to increase in length as smaller and smaller scales are included.

CONCLUSION

The preceding observations reveal that D_c is representative of the structural variability observed along the profile, whereas D_p is not. Wu (1995) offers a useful tool for estimating the contribution of several orders of folds to total bed-length shortening. Wu (1993) attempts to resolve differences between D_p and D_c through magnification of structural relief. However, there is no reason to assume that the two methods produce the same fractal dimension; D_p is not uniquely related to profile length. Attempts to resolve differences between D_p and D_c will lead to significant misrepresentation of bed-length shortening.

The compass dimension (D_c) is directly related to the length of the deformed layer. The compass method provides a means to predict bed-length shortening that incorporates higher order folds estimated from a few fold orders observed on a regional scale cross section. The geologist has to decide whether inclusion of finer scale structure is appropriate, since it is possible to obtain any percentage of shortening simply by including smaller and smaller order structures. The result is meaningless if structures at finer scales are not present. It may also happen that fractal interrelationships may extend only over two or three fold orders, and that the fractal dimension may change from one range of scales to another.

Acknowledgements—Comments of D. J. Sanderson were helpful and much appreciated. This work was supported through USDOE grant DE-FG21-95MC32158.

REFERENCES

Brown, S. (1987) A note on the description of surface roughness using fractal dimension. *Geophysical Research Letters* **14**, 1085–1098.

Dean, S., Kulander, B. and Lessing, P. (1985) Geology of the Capon Springs, Mountain Falls, Wardensville, Woodstock, and Yellow Spring Quadrangles, Hampshire and Hardy Counties, West Virginia. *West Virginia Geological and Economic Survey*, Map-WV26, Morgantown, WV.

Dunne, W. and Ferrill, D. (1995) Fractal strain distribution and its implications for cross-section balancing: Discussion. *Journal of Structural Geology* **17**, 757–760.

Mandelbrot, B. B. (1985) Self-affine fractals and fractal dimension. *Physica Scripta* **32**, 257–260.

Oppenheim, A. and Schafer, R. (1975) *Digital Signal Processing*. Prentice Hall, Englewood Cliffs.

Turcotte, D. L. (1992) *Fractals and Chaos in Geology and Geophysics*. Cambridge University Press, Cambridge.

Wilson, T. H. (1989) The Fourier analysis of structural cross sections. *Computer Oriented Geological Society Computer Contributions* **4**, 56–78.

Wong, R. (1987) Fractal surfaces in porous media. *American Institute of Physics* **154**, 304–318.

Wu, S. (1993) Fractal strain distribution and its implications on cross-section balancing. *Journal of Structural Geology* **15**, 1497–1507.

Wu, S. (1995) Fractal strain distribution and its implications for cross-section balancing: Reply. *Journal of Structural Geology* **17**, 761–764.